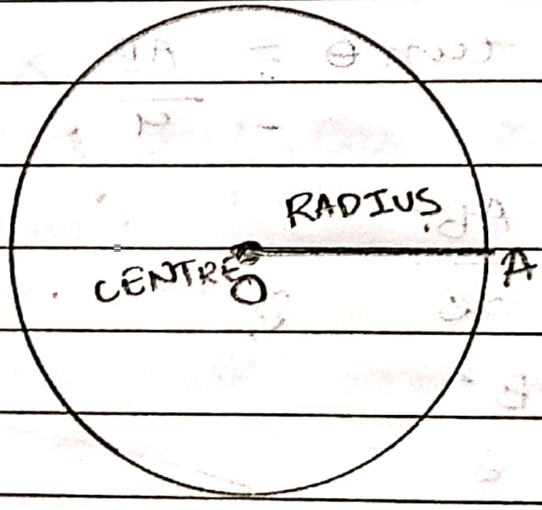
CIRCLES

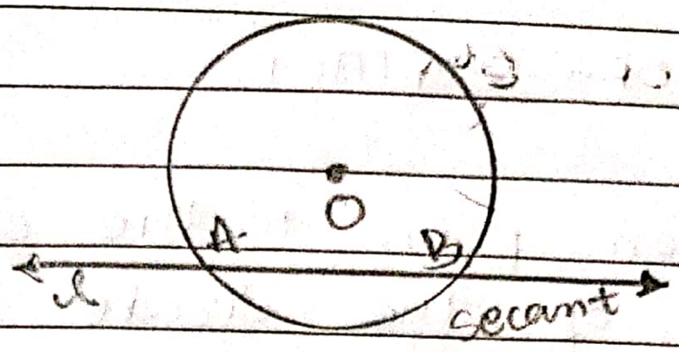
Circle is a collection or set or locus of points which are equidistant from fixed point.

The fixed point is called centre of circle and fixed distance is called radius of circle.

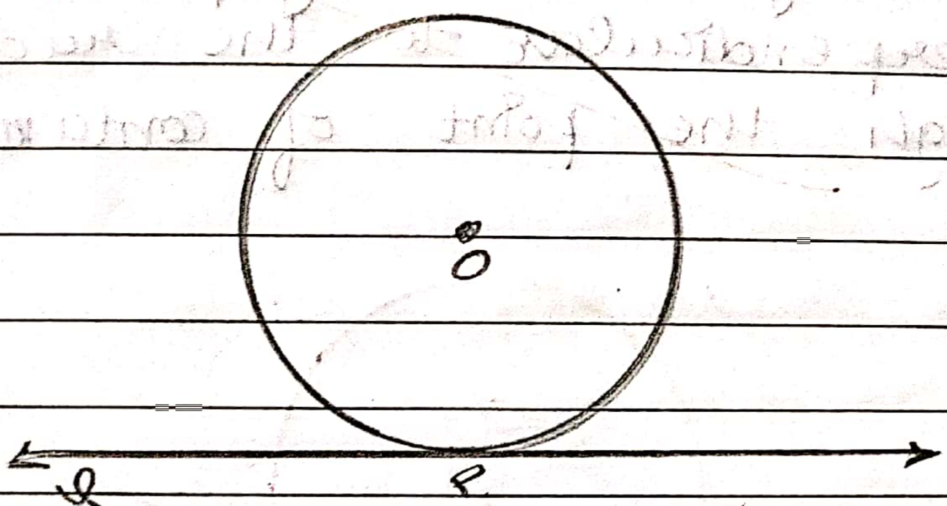


SECANT AND TANGENT OF CIRCLE

① A line which intersect a circle in two distinct points is called secant of circle.



A line which intersect the circle in one and only one point is called tangent to the circle.



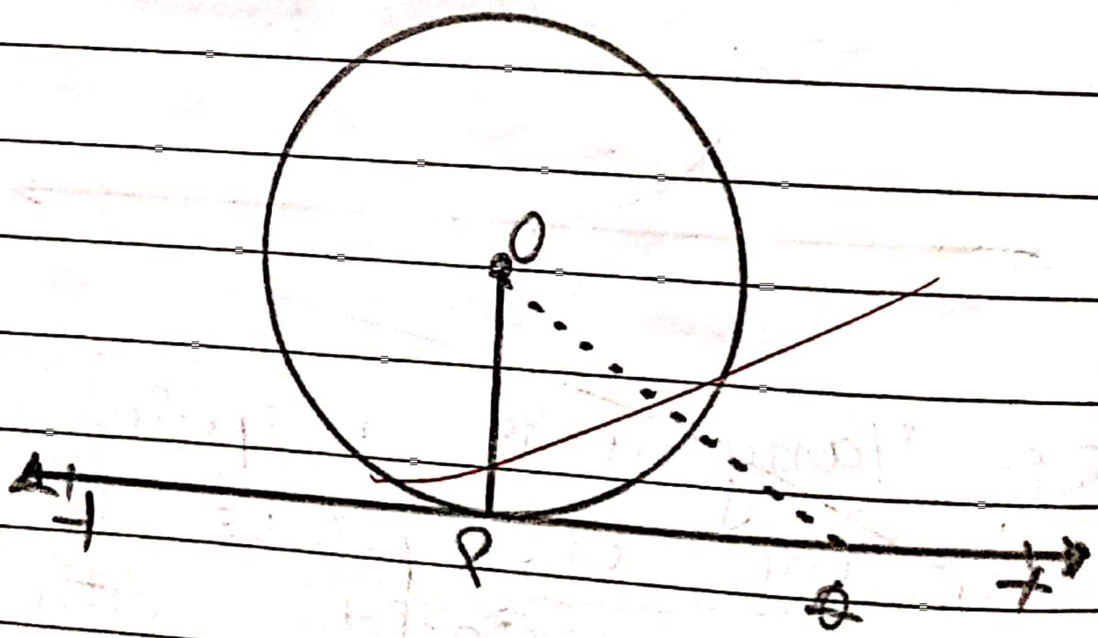
NOTE :- Tangent is a special case of secant when the end points of its corresponding chord are coincide with each other.

POINT OF CONTACT

A common point to the circle and tangent of the circle is called point of contact.

THEOREM 10.1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Proof :-

If we consider points other than P, such as Q.

We'll find that OP is the shortest distance.

$$OP > OQ$$

Thus OP will be the perpendicular to XY.

$$OP \perp XY$$

Hence proved.

(i) A tangent to a circle intersects it in one point (s).

(ii) A line intersecting a circle in two points is called a secant.

(iii) A circle can have two parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called point of contact.

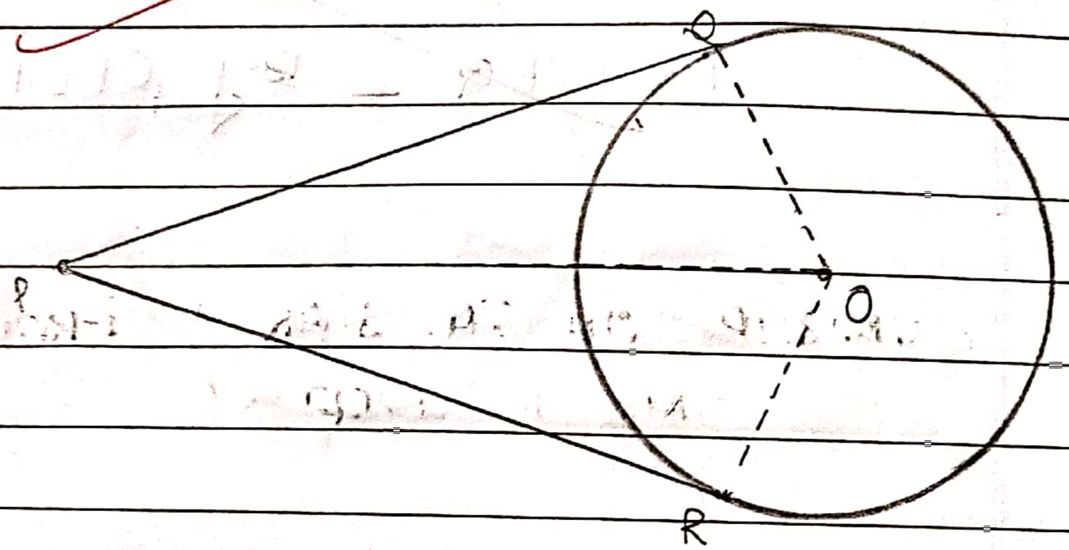
NUMBER OF TANGENTS FROM A POINT TO ON A CIRCLE

(1) There is no tangent to a circle passing through a point lying inside the circle.

(2) There is one and only one tangent to a circle passing through a point lying on a circle.

③ There are exactly two tangents to a circle passing through a point lying outside the circle.

THEOREM 10.2 :- The lengths of tangents drawn from an external point to a circle are equal.



PROOF :-

Consider that a circle with centre O and PQ and PR are the two tangents drawn from external point P to the circle.

We need to prove that PQ = PR.

Join OP, OQ, OR

By theorem,

$PQ \perp OQ$

$PR \perp OR$

$\therefore$ In right Δ's

ΔORP & ΔOQP

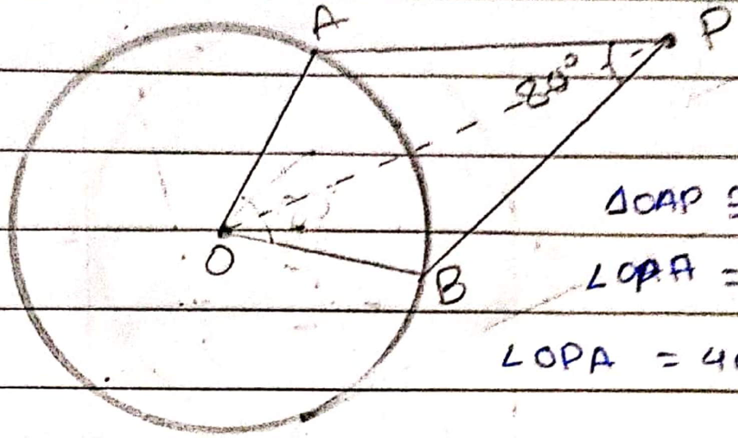
$OP = OP$ (common side)

$OR = OQ$ (radii of same circle)

$\Delta ORP \cong \Delta OQP$ by RHS

$PR = PQ$ — By CPCT

If tangents, - - - - - - is equal to :



In ΔOAP & ΔOBP,

$OA = OB$ (same radii)

$OP = OP$ (common)

$\Delta OAP \cong \Delta OBP$ (RHS Test)

$\angle OPA = \angle OPB$ (CPCT)

$\angle OPA = 40° = \angle OPB$

OA and OB be the radius

$OA \perp AP$ & $OB \perp BP$

$\angle OAP = 90° = \angle OBP$

In ΔOAP,

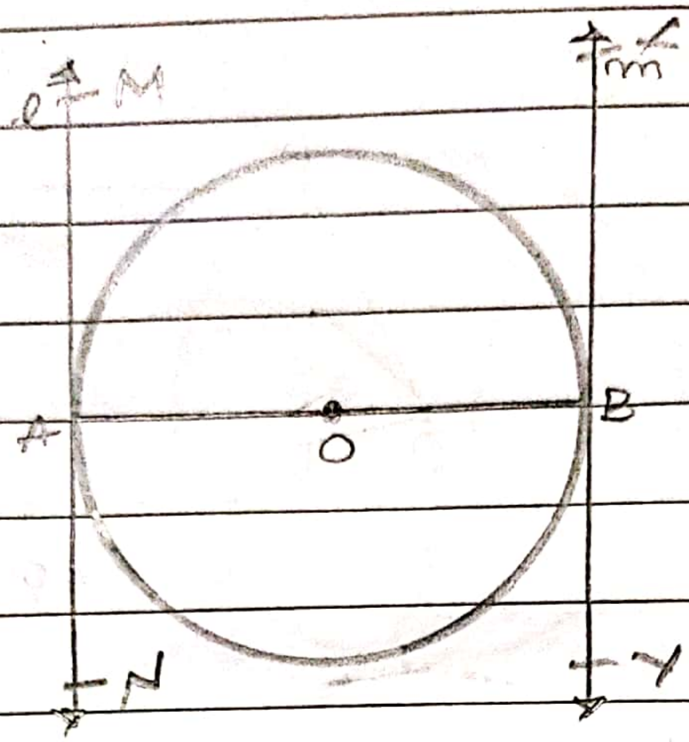
$\angle OAP + \angle APO + \angle POA = 180°$

$90° + 40° + \angle POA = 180°$

$130° + \angle POA = 180°$

$\angle POA = 50°$

Prove ------ are parallel.



AB is the ~~radius~~ diameter. OA & OB are the radius

XY & MN are the tangents.

OA ⊥ MN & OB ⊥ XY

∠OAN = 90°

∠OAM = 90°

∠YBO = 90°

∠OBX = 90°

∠OAN + ∠YBO = 180° (Interior angles)

∠OAM + ∠OBX = 180° (Interior angles)

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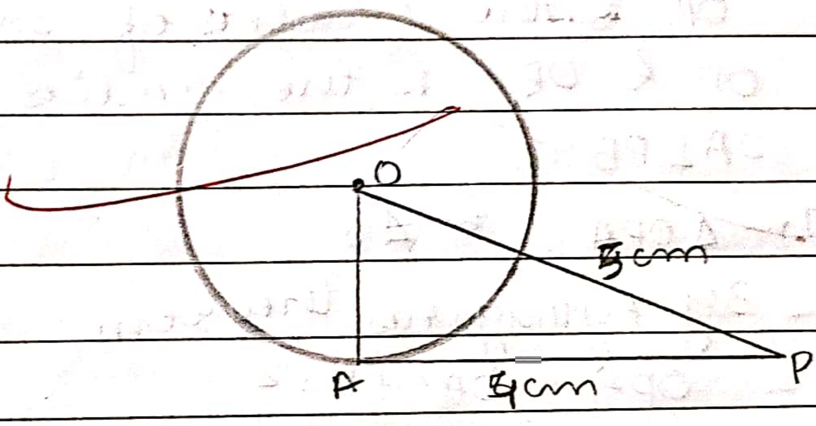
Interior angle
Sum of Interior angles is 180° so, Test
they are parallel to each other.

$\therefore XY \parallel MN$

Hence proved

6] The length - - - - the circle.

→



OA is the radius.

$OA \perp AP$ ($\perp$ to tangent)

By Pythagoras theorem,

In $\triangle OAP$,

$OP^2 = OA^2 + AP^2$

$(5)^2 = OA^2 + (4)^2$

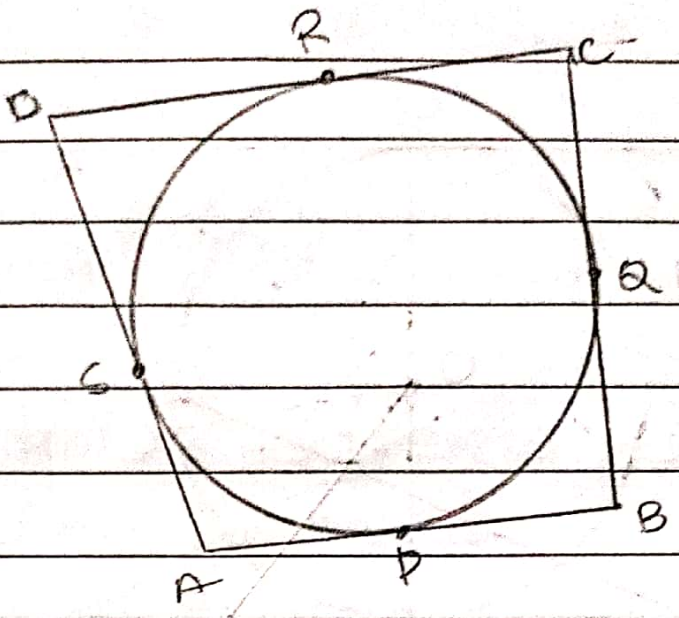
$25 = OA^2 + 16$

$OA^2 = 9$

$OA = 3$

$\therefore$ Radius of circle $= 3$

A quadrilateral - - - - - - - - $AB + CD = AD + BC$.



Tangents AP, AS, DS, DR, CR, CQ, QB, BP are drawn from external points A, B, C, D.

$AP = AS$ — (1)
$DS = DR$ — (2)
$CR = CQ$ — (3)
$BP = BQ$ — (4)

Length of tangents from an external points are equal.

Adding equations,

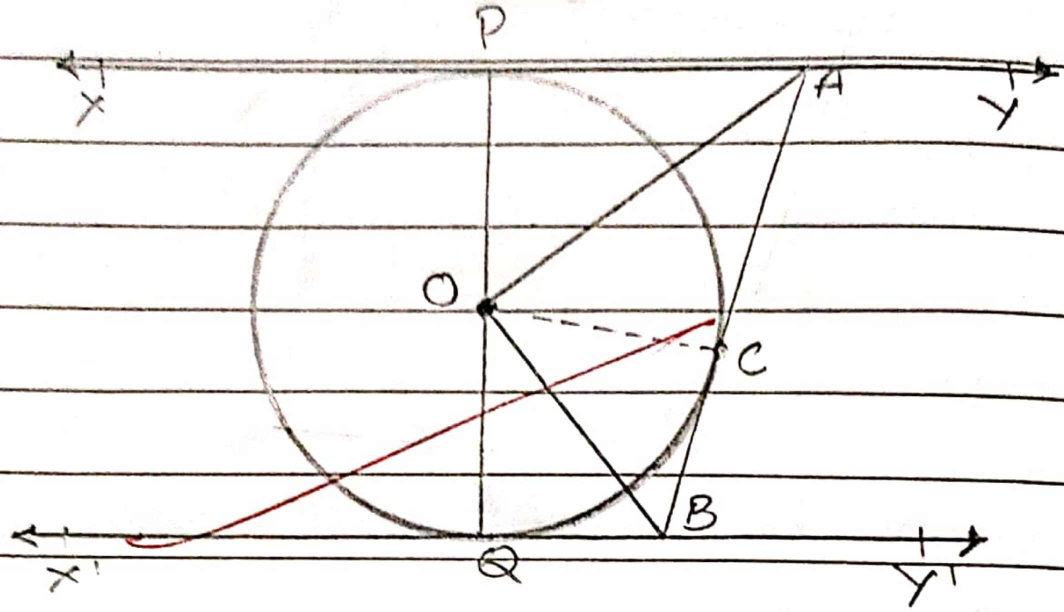
$AP + BP + CR + DR = AS + BQ + CQ + DS$

$AB + CD = BC + AD$

$AB + CD = AD + BC$

Hence proved

In fig ———————— $\angle AOB = 90°$.



Join OC

$OP \perp PA$
$OQ \perp BQ$
$OC \perp AB$
} Tangent perpendicular radius

$\therefore \angle OPA = \angle OQB = \angle OCA = \angle OCB = 90°$

In $\triangle OPA$ & $\triangle OCA$,

$OA = OA$ (common)

$OP = OC$ (Radii of same circle)

$\triangle OPA \cong \triangle OCA$ By RHS

$\angle POA = \angle AOC$ CPCT

Let $\angle POA = \angle AOC = x$ —— (1)

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Similarly,

$\triangle OQB \cong \triangle OCB$ by RHS $\angle QOB = \angle COB$ (CPCT)

$\therefore \angle QOB = \angle COB = y$

$\angle POA + \angle AOC + \angle BOC + \angle BOQ = 180^\circ$ (Linear pair)

$x + x + y + y = 180^\circ$ (from ① & ②)

$2x + 2y = 180^\circ$

$2(x+y) = 180^\circ$

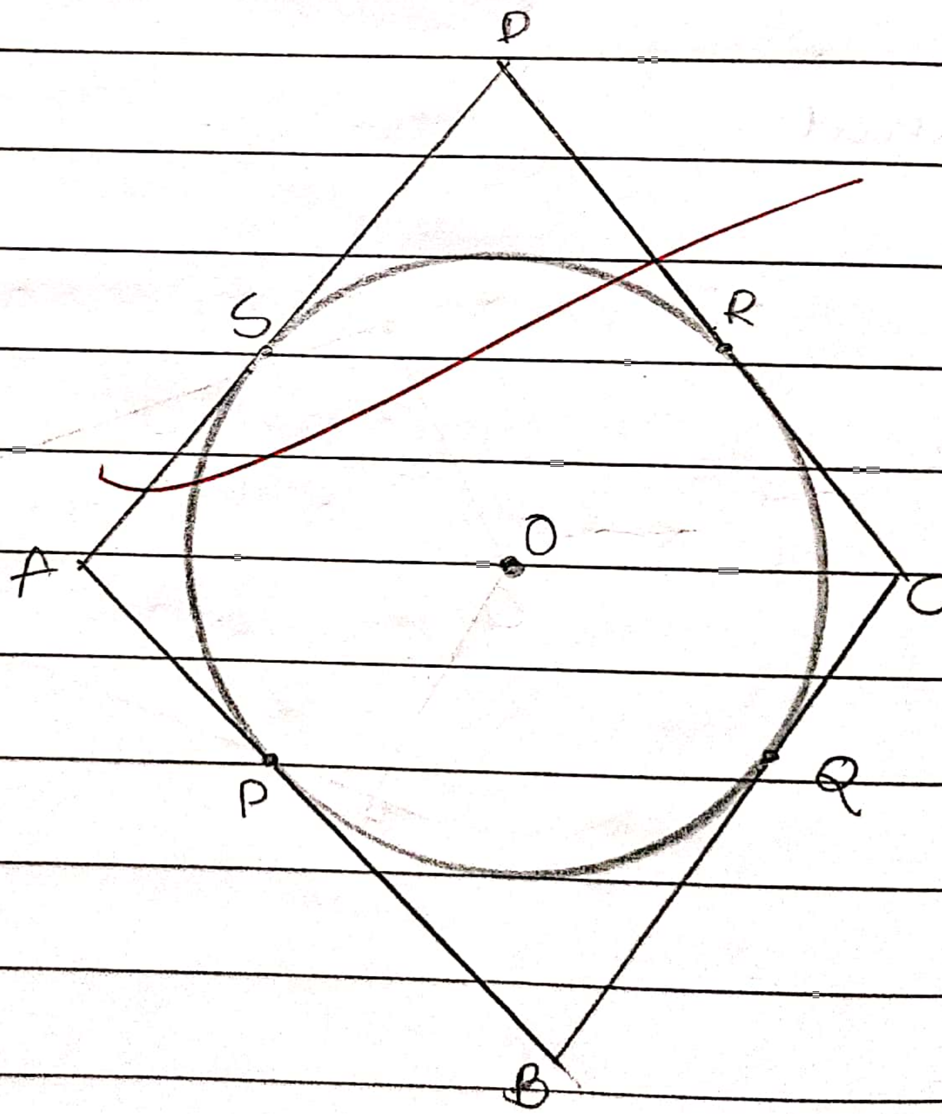
$x + y = 90^\circ$

$\angle AOC + \angle BOC = 90^\circ$

$\angle AOB = 90^\circ$

Hence proved

Prove - - - - - rhombus.



→ ABCD is a parallelogram.

$\therefore AB = CD$ — (1)

$BC = AD$ — (2)

$$\left.\begin{aligned} DR &= DS \\ CR &= CQ \\ BP &= BQ \\ AP &= AS \end{aligned}\right\} \text{Tangents from external point}$$

Adding all these,

$DR + CR + BP + AP = DS + CQ + BQ + AS$

$DR + CR + BP + AP = DS + AS + CQ + BQ$

$CD + AB = AD + BC$ — (3)

Putting value of (1) & (2)

$2AB = 2BC$

$AB = BC$ — (4)

Comparing (1), (3) & (4) we get,

$AB = BC = CD = DA$

$\therefore$ ABCD is a rhombus.

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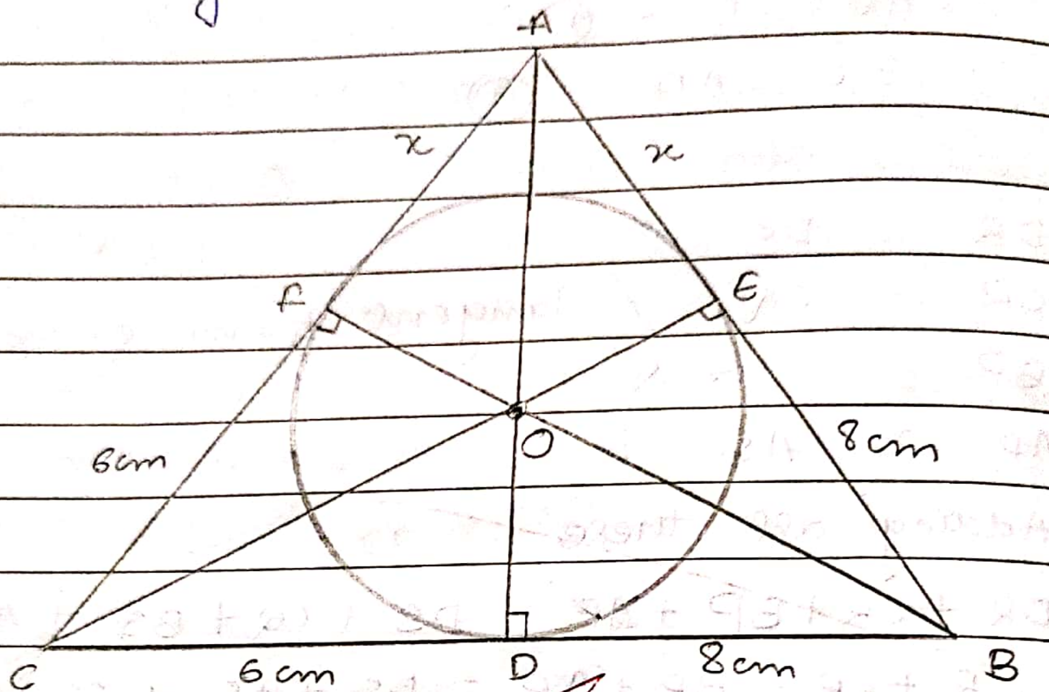
12) A triangle ABC - - - - - - - - - AB and AC.



In ΔABC,

$CF = CD = 6cm$ }
$BE = BD = 8cm$ } Tangents from external point
$AE = AF = x\ cm$ }

$AB = AE + EB = x + 8$

$BC = BD + DC = 8 + 6 = 14$

$CA = CF + FA = 6 + x$

Semiperimeter s,

$2s = AB + BC + CA$

$x = x + 8 + 14 + 6 + x$

$= 28 + 2x$

$s = 14 + x$

By Heron's formula,
In $\triangle ABC$

$$ar(ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{(14+x)(14+x-14)(14+x-x-6)(14+x-x-8)}$$
$$= \sqrt{(14+x)(x)(8)(6)}$$
$$= \sqrt{(14+x)48x} \quad \text{—— (1)}$$

$$ar(\cancel{ABC}\,COB) = \frac{1}{2} \times BC \times AD$$
$$= \frac{1}{2} \times 14 \times 4$$
$$= 28$$

$$ar(AOB) = \frac{1}{2} \times AB \times OE$$
$$= \frac{1}{2} \times 8+x \times 4$$
$$= 16+2x$$

$$ar(AOC) = \frac{1}{2} \times AC \times OF$$
$$= \frac{1}{2} \times 6+x + 4$$
$$= 12+2x$$

$ar(ABC) = ar(COB) + ar(AOB) + ar(AOC)$

$= 28 + 16 + 2x + 14 + 2x$

$= 56 + 4x$ — (2)

Equating (1) & (2)

$\sqrt{(14+x)48x} = 56 + 4x$

Squaring

$48x(14+x) = (56+4x)^2$

$48x = \frac{[4(14+x)]^2}{(14+x)}$

$48x = 16(14+x)$

$48x = 224 + 16x$

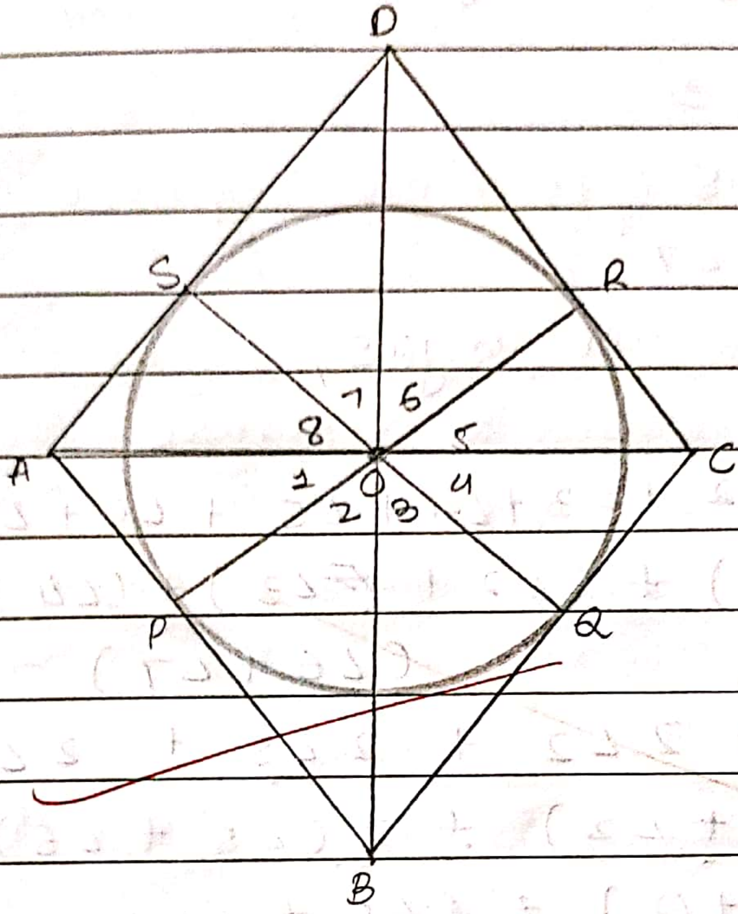
$32x = 224$

$x = 7\text{ cm}$

$\therefore AB = x+8 = 7+8 = 15\text{ cm}$

$AC = x+6 = 6+7 = 13\text{ cm}$

Prove that - - - - - - - - the circle.



Let ABCD be the quadrilateral.
P, Q, R, S are point of contact.
In Δ OAP & ΔOAS,
AP = AS (Tangent from external point)
OP = OS (Radii)
OA = OA (common)
$\Delta OAP \cong \Delta OAS$ (SSS criteria)
$\therefore \angle POA = \angle AOS$ By CPCT
$\angle 1 = \angle 8$

Similarly we get,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

Adding all angles,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$$

$$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$$

$$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^\circ$$

$$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^\circ$$

$$\angle AOB + \angle COD = 180^\circ$$

Similarly, $\angle BOC + \angle DOA = 180^\circ$

Hence proved.